

# Higher-Order Separation Logic For Probabilistic Programs




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# Probabilistic Programs

Useful

- Cryptography
- Randomized data structures/algos

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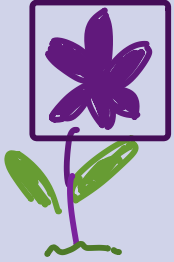
Limitations:

- Missing complex language features
  - higher-order
  - local state
- Modularity/compositional reasoning
- (Not mechanized/foundational)



## Our Approach

We construct probabilistic program logics on top of

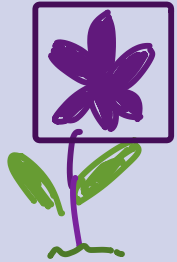


Iris, a

- Step-indexed
- Higher-order
- Machine-checked
- Separation Logic Framework

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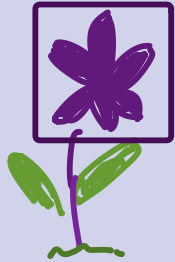
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- All formalized!

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Examples:

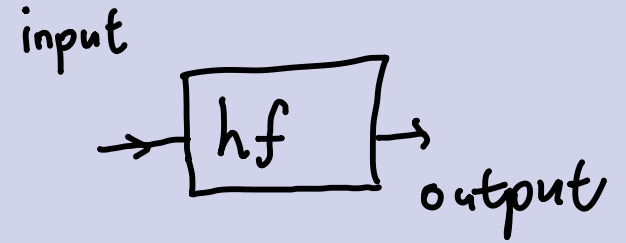
- Clutch: asynchronous couplings (POPL 2024)
- Caliper: termination preserving refinements (ICFP 2024)
- Eris: error bounds (ICFP 2024)
- Tachis: Expected cost (OOPSLA 2024)
- Approxis: approximate refinement (cond. accepted @ POPL 2025)

Something I  
contributed to!



# Case Study

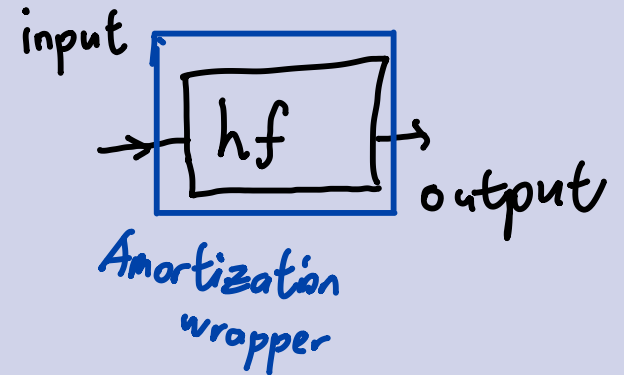
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Stage 2: An amortized hash with constant error

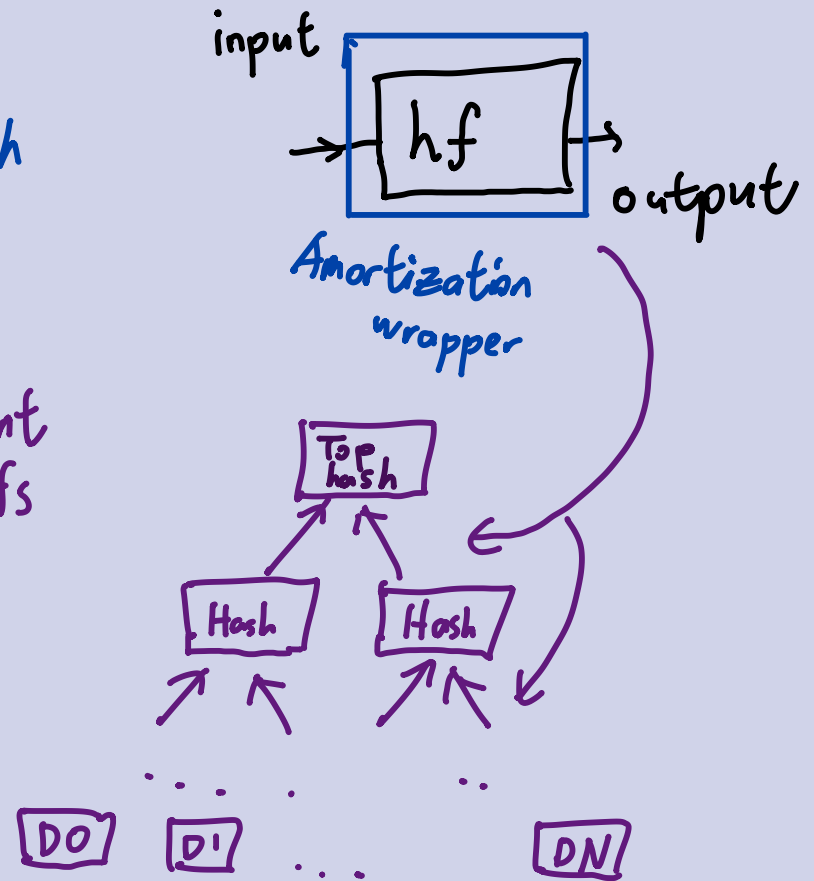


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Stage 1. An idealized hash function with error proportional to queries made

Stage 2: An amortized hash with constant error

Stage 3: A merkle tree with constant error bounds for validating proofs



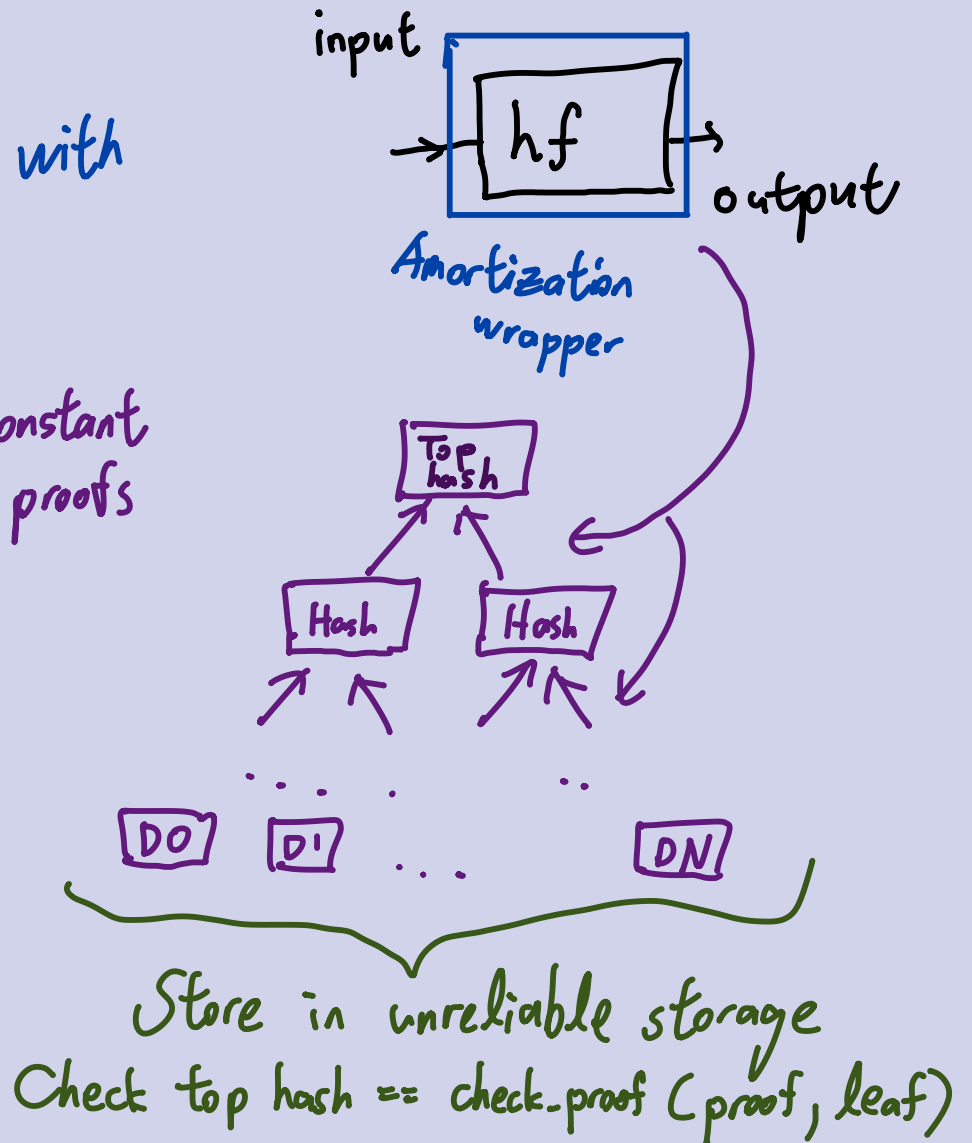
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Stage 1. An idealized hash function with error proportional to queries made

Stage 2: An amortized hash with constant error

Stage 3: A merkle tree with constant error bounds for validating proofs

Stage 4: Unreliable storage



## Conclusion

By formalizing our program logics with Iris,  
we can now verify many **large**, **complex**, and **EXCITING** examples,  
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Many more convincing examples:

- Security of ElGamal Public Key Encryption
- Amortized error bounds for hash map with resizing
- Expected cost of randomized meldable heap
- IND $\mathcal{A}$ -CPA security of symmetric encryption
- Contextual equivalences of B<sup>+</sup> tree sampling algorithms
- Many more ... =)

