Separation Logics for Probability, Concurrency, and *Security*

Kwing Hei Li (Heili)

Aarhus University

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Joint work with Alejandro Aguirre, Philipp G. Haselwarter,

Simon Oddershede Gergersen, Markus de Medeiros, Joseph Tassarotti, Lars Birkedal

Example: Password Storage

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\begin{aligned}
\mathsf{setpw}(m, u, p) &\triangleq \mathsf{set}\; m\; u\; p \\
\mathsf{checkpw}(m, u, p) &\triangleq \mathsf{match}\; \mathsf{get}\; m\; u\; \mathsf{with} \\
&\qquad \mathsf{Some}\; p' \Rightarrow p = p' \\
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We store passwords p of users u in a mutable map m.

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We store passwords p of users u in a mutable map m.
This is not secure!

Example: Password Storage *with hash*

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\mathsf{setpw}(m, u, p) &\triangleq \mathsf{set}\ m\ u\ (h(p)) \\
\mathsf{checkpw}(m, u, p) &\triangleq \mathsf{match}\ \mathsf{get}\ m\ u\ \mathsf{with} \\
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We now store the hash of the password instead.

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We now store the hash of the password instead.

People who use same passwords will have same hash stored!

Example: Password Storage *with hash and salt*

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\mathsf{setpw}(m, u, p) &\triangleq \mathsf{let\ salt} = \mathsf{rand}\ N\ \mathsf{in} \\
&\quad \mathsf{set}\ m\ u\ (\mathsf{salt}, h(\mathsf{salt} \cdot p)) \\
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We generate a salt (a random number from $0, \ldots, N$) for each call of setpw

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We now store both salt and result after hashing salt and password with hash function h

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Randomness occur in two places:

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Randomness occur in two places:

1. Generation of salt
2. Modelling hash function as random oracle

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randomness $\Rightarrow$ more complicated properties

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Observation 1:
randomness ⇒ more complicated properties

- checkpw with right password returns true
- checkpw with wrong password returns false with *high probability*
- password storage *appears random* to an outside observer

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Observation 2:
many complicated language features

- Dynamically allocated (potentially higher-order) mutable state

Example: Password Storage *with salt*

$$\mathsf{init} :: \mathsf{unit} \to \left(\begin{array}{l} \mathsf{setpw} : \mathsf{string} \to \mathsf{string} \to \mathsf{unit}, \\ \mathsf{checkpw} : \mathsf{string} \to \mathsf{string} \to \mathsf{bool} \end{array} \right)$$

$$\begin{array}{l} \mathsf{init} \triangleq \lambda\_.\ \mathsf{let}\ m = \mathsf{init}\,()\ \mathsf{in} \\ \quad \left(\begin{array}{l} \lambda u\, p.\ \mathsf{let\ salt} = \mathsf{rand}\ N\ \mathsf{in} \\ \qquad \mathsf{set}\ m\ u\ (\mathsf{salt}, h(\mathsf{salt} \cdot p)), \\ \lambda u\, p.\ \mathsf{match\ get}\ m\ u\ \mathsf{with} \\ \qquad \mathsf{Some}(\mathsf{salt}, x) \Rightarrow x = h(\mathsf{salt} \cdot p) \\ \qquad \mid \mathsf{None} \Rightarrow \mathsf{false} \\ \quad \mathsf{end} \end{array} \right) \end{array}$$

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- Higher order functions

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&\mathsf{init} \triangleq \lambda\_.\ \mathsf{let}\ m = \mathsf{init}\,()\ \mathsf{in} \\
&\qquad \left(\begin{aligned} &\lambda u\, p.\ \mathsf{let\ salt} = \mathsf{sample}\ N\ \mathsf{in} \\ &\qquad \mathsf{set}\ m\ u\ (\mathsf{salt}, h(\mathsf{salt} \cdot p)), \\ &\lambda u\, p.\ \mathsf{match\ get}\ m\ u\ \mathsf{with} \\ &\qquad\quad \mathsf{Some}(\mathsf{salt}, x) \Rightarrow x = h(\mathsf{salt} \cdot p) \\ &\qquad\quad \mid \mathsf{None} \Rightarrow \mathsf{false} \\ &\qquad \mathsf{end} \end{aligned} \right) \\
&\mathsf{sample}\ N \triangleq\ (\mathsf{rec}\ f\ \_ = \\
&\qquad\qquad \mathsf{let}\ x = \mathsf{rand\ MAX\ in} \\
&\qquad\qquad \mathsf{if}\ x \leqslant N\ \mathsf{then}\ x\ \mathsf{else}\ f\,()\ )\ ()
\end{aligned}
$$

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- Unbounded looping

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\mathsf{client} &\triangleq \mathsf{let}\ (\mathsf{setpw}, \mathsf{checkpw}) = \mathsf{init}\,()\ \mathsf{in} \\
&\quad \big(\mathsf{setpw}(u_1, p_1) \mathbin{|||} \mathsf{setpw}(u_2, p_2)\big); \\
&\quad \mathsf{checkpw}(u_1, p_2)
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$$\mathsf{init} \triangleq \lambda\_.\ \mathsf{let}\ m = \mathsf{init}\,()\ \mathsf{in}$$

$$\left(\begin{array}{l} \lambda u\, p.\ \mathsf{let\ salt} = \mathsf{read}\ \textbf{/dev/random}\ \mathsf{in} \\ \qquad \mathsf{set}\ m\ u\ (\mathsf{salt}, h(\mathsf{salt} \cdot p)), \\ \lambda u\, p.\ \mathsf{match\ get}\ m\ u\ \mathsf{with} \\ \qquad \mathsf{Some}(\mathsf{salt}, x) \Rightarrow x = h(\mathsf{salt} \cdot p) \\ \qquad \mid \mathsf{None} \Rightarrow \mathsf{false} \\ \quad \mathsf{end} \end{array} \right)$$

$\mathsf{generator} \triangleq$ *repeatedly writes random bits into* ***/dev/random***

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- Dynamically allocated (potentially higher-order) mutable state
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- Unbounded looping
- Concurrency in client & implementation...

Verifying real-world security programs $\Rightarrow$ Reasoning about probabilistic properties $+$ Using complicated language features

Proving complicated probabilistic properties

Various prior work on verifying probabilistic programs:

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- Refinement based approaches...

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- Refinement based approaches...

Though they have various limitations, e.g. no shared state, higher-order functions,

Iris

Iris is a higher-order concurrent separation logic framework, formalized in *Rocq*

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Used to verify programs with many *challenging features*, e.g. higher-order functions, unstructured concurrency

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However, less work on using Iris to prove *probabilistic* properties...

Logics developed

PhD goal: Develop *probabilistic extensions* of Iris for highly expressive languages

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	Unary	**Relational**
Sequential	Eris	Approxis
Concurrent	Coneris	Foxtrot

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Stage 1: develop Iris logics for sequential probabilistic programs

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Stage 1: develop Iris logics for sequential probabilistic programs

Stage 2: extend those logics to concurrent programs

Idealized collision-free hash

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\begin{aligned}
&\mathsf{let}\ x = h\ n\ \mathsf{in}\\
&\mathsf{let}\ y = h\ m\ \mathsf{in}\\
&(x, y)
\end{aligned}
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$$\left\{ \; m \neq n \; \right\} \quad \begin{array}{l} \text{let } x = h\; n \text{ in} \\ \text{let } y = h\; m \text{ in} \\ (x, y) \end{array} \quad \left\{ \; (x, y).\; x \neq y \; \right\}$$

Useful to model the hash function as a collision-free random oracle

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Hash is collision-free if different inputs map to different outputs

But this is not always true! Small probability of error!

Eris @ ICFP 2024

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- $\lightning(\varepsilon)$ asserts ownership of ε error credits, with $\varepsilon \in [0, 1]$
- Adequacy: $\{\lightning(\varepsilon)\}\, e\, \{v.\phi(v)\} \Rightarrow \mathrm{Pr}_{\mathsf{exec}\, e}[\neg\phi] \leqslant \varepsilon$

- *Eris* is a unary logic for proving error bounds of probabilistic programs
- **KEY IDEA: We internalize error as a separation logic resource, aka *error credit***
- $⚡(\varepsilon)$ asserts ownership of ε error credits, with $\varepsilon \in [0, 1]$
- Adequacy: $\{⚡(\varepsilon)\}\, e\, \{v.\phi(v)\} \Rightarrow \Pr_{\mathsf{exec}\, e}[\neg\phi] \leqslant \varepsilon$
- Flexible rules to "spend" error credits to avoid undesirable error results:

$$\frac{\textsc{ht-rand-list}}{\vdash \{⚡(\mathsf{length}(xs)/(N+1))\}\ \mathsf{rand}\ N\, \{n\ .\ n \notin xs\}}$$

Eris example: Hash

Idealized collision-free hash function

$$\left\{ \begin{array}{l} \mathsf{collFree}(\mathsf{h}) \ * \\ n \notin \mathrm{dom}\, h \ * \\ \lightning\left(\frac{|\mathrm{dom}\, h|}{2^S}\right) \end{array} \right\}$$

$$h\ n$$

$$\{v.\, \mathsf{collFree}(\mathsf{h})\}$$

Eris example: Hash

Idealized collision-free hash function

$\Downarrow$

Amortized idealized collision-free hash function

$$\left\{ \begin{array}{c} \mathsf{collFreeAm}(\mathsf{h}) \ast \\ n \notin \mathrm{dom}\, h \ast \\ |h| < M \ast \\ ⚡(E_{\mathrm{const}}) \end{array} \right\}$$

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Amortized hash specification used in verifying *Merkle tree* and *unreliable data storage system*

$$\mathsf{prf} \triangleq \lambda\_.\ \mathsf{rand}\, N$$

$$\mathsf{prp} \triangleq \begin{array}{l} \mathsf{let}\, l = \mathsf{ref}\, [\,]\ \mathsf{in} \\ \lambda\_.\ \mathsf{let}\, x = \mathsf{unif}\, (\{0, \ldots, N\} \setminus l)\ \mathsf{in} \\ \quad l \leftarrow x \cdot l; \\ \quad x \end{array}$$

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Used in security-related examples: PRP/PRF switching lemma and IND$-CPA security of an encryption scheme

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Approxis re-introduce error credits to the relational setting for proving *approximate refinements*

Used in security-related examples: PRP/PRF switching lemma and IND$-CPA security of an encryption scheme

Built a logical refinement relation for contextual refinement, used to prove correctness of a B+ tree sampling scheme

Logics for Concurrency and Probability

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We now extend them for concurrent probabilistic programs

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- Eris $\Rightarrow$ *Coneris* @ ICFP 2025
- Approxis $\Rightarrow$ *Foxtrot* (WIP)

Challenges in extending to Concurrency

These extensions to concurrency are non-trivial:

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2. Some rules in Approxis are unsound in Foxtrot

We need to redesign the model of the logics and introduce new logical facilities and proof techniques

Examples of Coneris and Foxtrot

- Modular specifications of *thread-safe* hashes

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- Modular specifications of *thread-safe* hashes
- Strict error bounds of *concurrent* Bloom filter
- Sodium sampling function:

$$
\begin{array}{l}
\lambda N.\ \text{if } N < 2 \text{ then } 0 \\
\qquad \text{else let min} = \text{MAX mod } N \text{ in} \\
\qquad \text{let } r = \text{ref } 0 \text{ in} \\
\qquad \left(\begin{array}{l} \text{rec} f\ \_ = r \leftarrow \text{rand}(\text{MAX} - 1); \\ \ \text{if } !r < \text{min then } f() \\ \ \text{else } (!r \text{ mod } N) \end{array} \right) ()
\end{array}
\quad \simeq_{\text{ctx}} \quad \lambda N.\ \text{if } N = 0 \text{ then } 0 \text{ else } \text{rand}(N - 1)
$$

Revisiting Password Storage

$$
\begin{array}{l}
\mathsf{init} \triangleq \lambda\_.\ \mathsf{let}\ m = \mathsf{init}\,()\ \mathsf{in} \\
\qquad \left(\begin{array}{l} \lambda u\, p.\ \mathsf{let\ salt} = \mathsf{sample}\ N\ \mathsf{in} \\ \qquad\quad \mathsf{set}\ m\ u\ (\mathsf{salt}, h(\mathsf{salt} \cdot p)), \end{array} \right. \\
\qquad\ \ \left. \begin{array}{l} \lambda u\, p.\ \mathsf{match\ get}\ m\ u\ \mathsf{with} \\ \qquad\quad \mathsf{Some}(\mathsf{salt}, x) \Rightarrow x = h(\mathsf{salt} \cdot p) \\ \qquad\quad \mid \mathsf{None} \Rightarrow \mathsf{false} \\ \qquad\ \mathsf{end} \end{array} \right)
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- checkpw with wrong password returns false with *high probability* $\Rightarrow$ Eris

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&\text{client} \triangleq \text{let } (\text{setpw}, \text{checkpw}) = \text{init}\,()\ \text{in} \\
&\qquad (\text{setpw}(u_1, p_1) \mid\mid\mid \text{setpw}(u_2, p_2)); \\
&\qquad \text{checkpw}(u_1, p_2)
\end{aligned}
$$

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Why? The schedulers are too powerful. (Well-known issue in various security models)

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Why? The schedulers are too powerful. (Well-known issue in various security models)

Can we develop logics for reasoning about more restricted schedulers?

Conclusion

Two challenges in verifying real-world security programs:

1. Complicated probabilistic properties
2. Programs use complicated language features

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Much success with implementing logics within *Iris*:

	Unary	**Relational**
Sequential	Eris	Approxis
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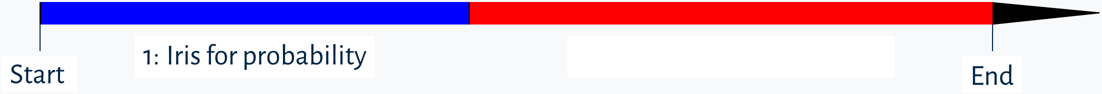
Future work:

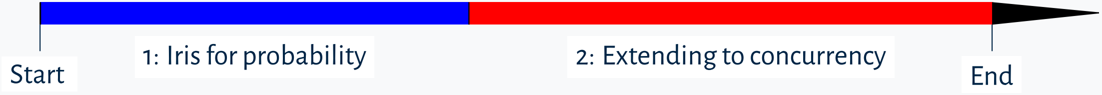
1. Improving concurrency model of Coneris and Foxtrot
2. Applying it to verify actual implementations of cryptographic libraries and protocols

APPENDIX

Timeline



Timeline


Start
1: Iris for probability
2: Extending to concurrency
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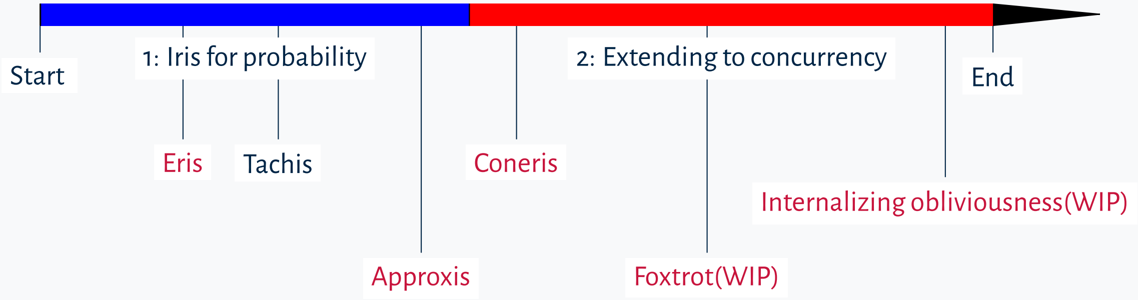

Start
1: Iris for probability
Eris
Tachis
Approxis
Coneris
2: Extending to concurrency
Foxtrot(WIP)
Internalizing obliviousness(WIP)
End

Eris rules

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$$\begin{tikzcd}
 & \boxed{0\,;\ \lightning(\mathcal{F}(0))} \\
\boxed{\mathsf{rand}\, 1\,;\ \lightning(\varepsilon)} \arrow[ur] \arrow[dr] & \\
 & \boxed{1\,;\ \lightning(\mathcal{F}(1))}
\end{tikzcd}$$

$$\frac{\mathcal{F}(0) + \mathcal{F}(1)}{2} \leqslant \varepsilon$$

Logical Relations in Approxis

We build a logical refinement relation in Approxis for proving contextual refinement

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You can assume ownership of some non-zero amount of error credits with the logical refinement relation!

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Used in proving correctness of a rejection sampling scheme from B+ tree (developed by Olken and Rotem 1989s)

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Coneris @ ICFP 2025

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- Inherits all the error credit rules of Eris
- Error credits can be placed in invariants!

$$
\begin{array}{l}
\{\lightning(1/16)\} \\
\qquad \mathsf{let}\ l = \mathsf{ref}\, 0\ \mathsf{in} \\
\qquad (\mathsf{faa}\ l\ (\mathsf{rand}\ 3) \mathrel{|||} \mathsf{faa}\ l\ (\mathsf{rand}\ 3))\,; \\
\qquad !\, l \\
\{v.v > 0\}
\end{array}
$$

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- We introduce the probabilistic update modality $\mid\!\rightsquigarrow$ to capture *randomized logical atomicity*
- Used to prove specification of a thread safe hash module and concurrent bloom filter (novel result)

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- Does not inherit all the rules of Approxis!

THIS-IS-UNSOUND

$$\frac{\kappa' \hookrightarrow_s (N,\vec{m}) \qquad \begin{array}{c}\kappa \hookrightarrow (N,\vec{n}) \\ \forall v.\, \kappa \hookrightarrow (N,\vec{n} +\!\!+ v) * \kappa' \hookrightarrow_s (N,\vec{m} +\!\!+ v) \mathrel{-\!\!*} \mathsf{rwp}\ e_1 \precsim e_2\ \{\Phi\}\end{array}}{\mathsf{rwp}\ e_1 \precsim e_2\ \{\Phi\}}$$

UNSOUND-HT-COUPLE-RAND-LBL-EXACT

$$\frac{\forall n \leqslant N.\, \{\kappa \hookrightarrow_s (N,\vec{n}\cdot[n])\}\, n\, \{\Phi\}}{\{\kappa \hookrightarrow_s (N,\vec{n})\}\ \mathsf{rand}\, N\, \{\Phi\}}$$

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Challenge: Model of Foxtrot is very different from that of Approxis.

Foxtrot examples

Algebraic theory:

$$(e_1 \oplus_p e_2) \oplus_q e_3 \simeq_{\mathsf{ctx}} e_1 \oplus_{pq} (e_2 \oplus_{\frac{q-pq}{1-pq}} e_3)$$

$$e_1 \mathbf{\ or\ } (e_2 \mathbf{\ or\ } e_3) \simeq_{\mathsf{ctx}} (e_1 \mathbf{\ or\ } e_2) \mathbf{\ or\ } e_3$$

$$e_1 \mathbf{\ or\ } (\mathsf{diverge}\,()) \simeq_{\mathsf{ctx}} e_1$$

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Libsodium random sampling implementation:

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Oblivious scheduler example

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